Michigan Merit Curriculum

Course/Credit Requirements

ALGEBRA II

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1 Credit

MICHIGAN Department of Education

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Welcome

This guide was developed to assist teachers in successfully implementing the Michigan Merit Curriculum. The identified content expectations and guidelines provide a successful framework for designing curriculum, assessments and relevant learning experiences for students. Through the collaborative efforts of Governor Jennifer M. Granholm, the State Board of Education, and the State Legislature, these landmark state graduation requirements are being implemented to give Michigan students the knowledge and skills to succeed in the 21st Century and drive Michigan's economic success in the global economy. Working together, teachers can explore varied pathways to help students demonstrate proficiency in meeting the content expectations and guidelines.

Curriculum Unit Design

One of the ultimate goals of teaching is for students to acquire transferable knowledge. To accomplish this, learning needs to result in a deep understanding of content and mastery level of skills. As educational designers, teachers must use both the art and the science of teaching. In planning coherent, rigorous instructional units of study, it is best to *begin with the end in mind*.

Engaging and effective units include

- appropriate content expectations
- students setting goals and monitoring own progress
- a focus on big ideas that have great transfer value
- focus and essential questions that stimulate inquiry and connections
- identified valid and relevant skills and processes
- purposeful real-world applications
- relevant and worthy learning experiences
- varied flexible instruction for diverse learners
- research-based instructional strategies
- explicit and systematic instruction
- adequate teacher modeling and guided practice
- substantial time to review or apply new knowledge
- opportunities for revision of work based on feedback
- student evaluation of the unit
- culminating celebrations

Relevance

Instruction that is clearly relevant to today's rapidly changing world is at the forefront of unit design. Content knowledge cannot by itself lead all students to academic achievement. Classes and projects that spark student interest and provide a rationale for why the content is worth learning, enable students to make connections between what they read and learn in school, their lives, and their futures. An engaging and effective curriculum provides opportunities for exploration and exposure to new ideas. Real-world learning experiences provide students with opportunities to transfer and apply knowledge in new diverse situations.

Student Assessment

The assessment process can be a powerful tool for learning when students are actively involved in the process. Both assessment *of* learning and assessment *for* learning, are essential. Reliable formative and summative assessments provide teachers with information they need to make informed instructional decisions that are more responsive to students' needs. Engagement empowers students to take ownership of their learning and builds confidence over time.

Sound assessments:

- align with learning goals
- vary in type and format
- use authentic performance tasks
- use criteria scoring tools such as rubrics or exemplars
- allow teachers and students to track growth over time
- validate the acquisition of transferable knowledge
- give insight into students' thinking processes
- cause students to use higher level thinking skills
- address guiding questions and identified skills and processes
- provide informative feedback for teachers and students
- ask students to reflect on their learning

High School Content Expectation Codes

To allow for ease in referencing expectations each mathematics expectation has been coded by strand, standard, topic, and expectation. For example:

A1.2.3

- **A:** Algebra and Functions strand
- **A1:** Standard 1 of the Algebra and Functions strand
- **A1.2:** Topic 2 in Standard A1
- **A.1.2.3:** 3rd expectation in the 2nd topic of Standard A1

Organizational Structure

STRAND 1 **Quantitative Literacy and Logic (L)**	**STRAND 2** **Algebra and Functions (A)**
STANDARDS (and number of core expectations in each standard)	
L1: Reasoning About Numbers, Systems and Quantitative Situations (9) **L2: Calculation, Algorithms, and Estimation** (9) **L3: Measurement and Precision** (5) **L4: Mathematical Reasoning, Logic, and Proof** (10)	**A1: Expressions, Equations, and Inequalities** (16) **A2: Function** (39) **A3: Mathematical Modeling** (3)
Recommended Quantitative Literacy and Logic Expectations (3)	*Recommended Algebra and Functions Expectations (5)*

STRAND 3 **Geometry and Trigonometry (G)**	**STRAND 4** **Statistics and Probability (S)**
STANDARDS (and number of core expectations in each standard)	
G1: Figures and Their Properties (29) **G2: Relationships Between Figures** (10) **G3: Transformations of Figures in the Plane** (5)	**S1: Univariate Data—Examining Distributions** (9) **S2: Bivariate Data—Examining Relationships** (6) **S3: Samples, Surveys, and Experiments** (3) **S4: Probability Models and Probability Calculation** (4)
Recommended Geometry and Trigonometry Expectations (3)	*Recommended Statistics and Probability Expectations (6)*

Recommended Expectations

At the end of each strand, a set of recommended expectations may be listed. These extensions represent content that is desirable and valuable for all students, but attention to these items should not displace or dilute the curricular emphasis of any of the core expectations. Teachers are encouraged to incorporate the recommended expectations into their instruction when their students have a solid foundation and are ready for enrichment or advanced learning. Recommend expectations will not be tested on the Michigan Merit Exam or on future high school subject credit assessments.

Coding Note: Recommended expectations have an * preceding their code

Organization of this Document

In the Mathematics credit requirement documents the expectations are organized by strand and standard underneath topic headings. The organization in no way implies an instructional sequence. Curriculum personnel or teachers are encouraged to organize these topics and expectations in a manner that encourages connections between strands and among topics with a strand.

Introduction to Algebra II

The increasing use of quantitative methods in all disciplines has made algebra the fundamental tool for mathematical applications. Algebraic thinking is learned most effectively when it is studied in the context of applications, both mathematical and real-world. These applications reveal the power of algebra to model real problems and to generalize new situations. Algebra is not only a theoretical tool for analyzing and describing mathematical relationships, but it is also a powerful tool for the mathematical modeling and solving of real-world problems. These problems can be found all around us: the workplace, the sciences, technology, engineering and mathematics.

Algebra II Goal Statement

The goal of Algebra II is to build upon the concepts taught in Algebra I and Geometry while adding new concepts to the students' repertoire of mathematics. In Algebra I, students studied the concept of functions in various forms such as linear, quadratic, polynomial, and exponential. Algebra II continues the study of exponential and logarithmic functions and further enlarges the catalog of function families to include rational and trigonometric functions. In addition to extending the algebra strand, Algebra II will extend the numeric and logarithmic ideas of accuracy, error, sequences, and iteration. The topic of conic sections fuses algebra with geometry. Students will also extend their knowledge of univariate and bivariate statistical applications.

It is the purpose of Algebra II to give the students a rigorous understanding of the expectations included within it. It is also the goal of this model to help students see the connections in the mathematics that they have already learned. For example, students will not only gain an in-depth understanding of circular trigonometry, but will also understand its connections to triangular trigonometry. Connections between trigonometric modeling of cyclic events and the concepts embedded within bivariate modeling with the proper use of statistical techniques will also be made.

Throughout Algebra I & II, students will experience mathematics generally, and algebra in particular, not only as the theoretical study of mathematical patterns and relationships but also as a language that allows us to make sense of mathematical symbols. Finally, students will develop an understanding that algebraic thinking is an accessible and powerful tool that can be used to model and solve real-world problems.

Algebra II Content Expectations Outline

STANDARD L1: REASONING ABOUT NUMBERS, SYSTEMS, AND QUANTITATIVE SITUATIONS

- L1.2 Representations and Relationships
- L1.3 Counting and Probabilistic Reasoning

STANDARD L2: CALCULATION, ALGORITHMS, AND ESTIMATION

- L2.1 Calculation Using Real and Complex Numbers
- L2.2 Sequences and Iteration

STANDARD L3: MEASUREMENT AND PRECISION

- L3.2 Understanding Error

STANDARD A1: EXPRESSIONS, EQUATIONS, AND INEQUALITIES

- A1.1 Construction, Interpretation, and Manipulation of Expressions (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)
- A1.2 Solutions of Equations and Inequalities (linear, exponential, logarithmic, quadratic, power, polynomial, rational, and trigonometric)

STANDARD A2: FUNCTIONS

A2.3	Families of Functions (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)
A2.5	Exponential and Logarithmic Functions
A2.9	Rational Functions
A2.10	Trigonometric Functions

STANDARD A3: MATHEMATICAL MODELING

A3.1	Models of Real-world Situations Using Families of Functions

STANDARD G1: FIGURES AND THEIR PROPERTIES

G1.7	Conic Sections and Their Properties

STANDARD S1: UNIVARIATE DATA – EXAMINING DISTRIBUTIONS

S1.1	Producing and Interpreting Plots
S1.2	Measures of Center and Variation
S1.3	The Normal Distribution

STANDARD S3: SAMPLES, SURVEYS, AND EXPERIMENTS

S3.1	Data Collection and Analysis

STANDARD S4: PROBABILITY MODELS AND PROBABILITY CALCULATION

S4.1	Probability
S4.2	Application and Representation

CONTENT EXPECTATIONS FOR ALGEBRA II

STANDARD L1: REASONING ABOUT NUMBERS, SYSTEMS, AND QUANTITATIVE LITERACY

L1.2 Representations and Relationships

L1.2.1 Use mathematical symbols (e.g., interval notation, set notation, summation notation) to represent quantitative relationships and situations.

L1.3 Counting and Probabilistic Reasoning

L1.3.1 Describe, explain, and apply various counting techniques (e.g., finding the number of different 4-letter passwords; permutations; and combinations); relate combinations to Pascal's triangle; know when to use each technique.

L1.3.2 Define and interpret commonly used expressions of probability (e.g., chances of an event, likelihood, odds).

L1.3.3 Recognize and explain common probability misconceptions such as "hot streaks" and "being due."

STANDARD L2: CALCULATION, ALGORITHMS, AND ESTIMATION

L2.1 Calculation Using Real and Complex Numbers

L2.1.6 Recognize when exact answers aren't always possible or practical; use appropriate algorithms to approximate solutions to equations (e.g., to approximate square roots).

L2.2 Sequences and Iteration

L2.2.1 Find the nth term in arithmetic, geometric, or other simple sequences.

L2.2.2 Compute sums of finite arithmetic and geometric sequences.

L2.2.3 Use iterative processes in such examples as computing compound interest or applying approximation procedures.

STANDARD L3: MEASUREMENT AND PRECISION

L3.2 Understanding Error

L3.2.1 Determine what degree of accuracy is reasonable for measurements in a given situation; express accuracy through use of significant digits, error tolerance, or percent of error; describe how errors in measurements are magnified by computation; recognize accumulated error in applied situations.

L3.2.2 Describe and explain round-off error, rounding, and truncating.

L3.2.3 Know the meaning of and interpret statistical significance, margin of error, and confidence level.

RECOMMENDED:

***L1.2.5** Read and interpret representations from various technological sources, such as contour or isobar diagrams.

***L2.1.7** Understand the mathematical bases for the differences among voting procedures.

***L2.2.4** Compute sums of infinite geometric sequences.

STANDARD A1: EXPRESSIONS, EQUATIONS AND INEQUALITIES

A1.1 Construction, Interpretation, and Manipulation of Expressions (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)

A1.1.4 Add, subtract, multiply, and simplify polynomials and rational expressions (e.g., multiply $(x - 1)$ $(1 - x^2 + 3)$; simplify $\frac{9x - x^3}{x + 3}$.

A1.1.5 Divide a polynomial by a monomial.

A1.2 Solutions of Equations and Inequalities (linear, exponential, logarithmic, quadratic, power, polynomial, rational, and trigonometric)

A1.2.5 Solve polynomial equations and equations involving rational expressions (e.g. solve $-2x(x^2 + 4x+3) = 0$; solve $x - \frac{1}{x+6} = 3$), and justify steps in the solution.

A1.2.7 Solve exponential and logarithmic equations (e.g., $3(2^x) = 24$), $2\ln(x + 1) = 4$), and justify steps in the solution.

A1.2.9 Know common formulas (e.g., slope, distance between two points, quadratic formula, compound interest, distance = rate · time), and apply appropriately in contextual situations.

A1.2.10 Use special values of the inverse trigonometric functions to solve trigonometric equations over specific intervals (e.g., $2\sin x - 1 = 0$ for $0 \le x \le 2\pi$).

STANDARD A2: FUNCTIONS

A2.3 Families of Functions (linear, quadratic, polynomial, rational, power, exponential, logarithmic, and trigonometric)

A2.3.3 Write the general symbolic forms that characterize each family of functions (e.g., $f(x) = A_0a^x$; $f(x) = A\sin Bx$).

A2.5 Exponential and Logarithmic Functions

A2.5.2 Interpret the symbolic forms and recognize the graphs of exponential and logarithmic functions (e.g., $f(x) = 10^x$, $f(x) = \log x$, $f(x) = e^x$, $f(x) = \ln x$).

A2.5.3 Apply properties of exponential and logarithmic functions (e.g., $a^{x+y} = a^xa^y$; $\log(ab) = \log a + \log b$).

A2.9 Rational Functions

A2.9.1 Write the symbolic form and sketch the graph of simple rational functions.

A2.9.2 Analyze graphs of simple rational functions (e.g., $f(x) = \frac{2x+1}{x-1}$; $g(x) = \frac{x}{x^2-4}$) and understand the relationship between the zeros of the numerator and denominator and the function's intercepts, asymptotes, and domain.

A2.10 Trigonometric Functions

A2.10.1 Use the unit circle to define sine and cosine; approximate values of sine and cosine (e.g., $sin\ 3$, or cos 0.5); use sine and cosine to define the remaining trigonometric functions; explain why the trigonometric functions are periodic.

A2.10.2 Use the relationship between degree and radian measures to solve problems.

A2.10.3 Use the unit circle to determine the exact values of sine and cosine, for integer multiples of $\pi/6$ and $\pi/4$.

A2.10.4 Graph the sine and cosine, functions; analyze graphs by noting domain, range, period, amplitude, location of maxima and minima, and asymptotes.

A2.10.5 Graph transformations of basic trigonometric functions (involving changes in period, amplitude, phase, and midline) and understand the relationship between constants in the formula and the transformed graph.

STANDARD A3: MATHEMATICAL MODELING

A3.1 Models of Real-world Situations Using Families of Functions

Example: An initial population of 300 people grows at 2% per year. What will the population be in 10 years?

A3.1.1 Identify the family of function best suited for modeling a given real-world situation (e.g., quadratic functions for motion of an object under the force of gravity; exponential functions for compound interest; trigonometric functions for periodic phenomena. *In the example above, recognize that the appropriate general function is exponential* ($P = P_0 a^t$)

A3.1.2 Adapt the general symbolic form of a function to one that fits the specifications of a given situation by using the information to replace arbitrary constants with numbers. *In the example above, substitute the given values* $P_0 = 300$ *and* $a = 1.02$ *to obtain* $P = 300(1.02)^t$.

A3.1.3 Using the adapted general symbolic form, draw reasonable conclusions about the situation being modeled. *In the example above, the exact solution is 365.698, but for this problem an appropriate approximation is 365.*

RECOMMENDED:

***A1.1.7** Transform trigonometric expressions into equivalent forms using basic identities such as:
$\sin^2\theta + \cos^2\theta = 1$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\tan^2\theta + 1 = \sec^2\theta$

***A2.2.4** If a function has an inverse, find the expression(s) for the inverse.

***A2.2.5** Write an expression for the composition of one function with another; recognize component functions when a function is a composition of other functions.

***A2.2.6** Know and interpret the function notation for inverses and verify that two functions are inverses using composition.

STANDARD G1: FIGURES AND THEIR PROPERTIES

G1.7 Conic Sections and Their Properties

G.1.7.1 Find an equation of a circle given its center and radius; given the equation of a circle, find its center and radius.

G1.7.2 Identify and distinguish among geometric representations of parabolas, circles, ellipses, and hyperbolas; describe their symmetries, and explain how they are related to cones.

G1.7.3 Graph ellipses and hyperbolas with axes parallel to the x- and y-axes, given equations.

***G1.7.4** Know and use the relationship between the vertices and foci in an ellipse, the vertices and foci in a hyperbola, and the directrix and focus in a parabola; interpret these relationships in applied contexts.

STANDARD S1: UNVARIATE DATA-EXAMINING DISTRIBUTIONS

S1.1 Producing and Interpreting Plots

S1.1.1 Construct and interpret dot plots, histograms, relative frequency histograms, bar graphs, basic control charts, and box plots with appropriate labels and scales; determine which kinds of plots are appropriate for different types of data; compare data sets and interpret differences based on graphs and summary statistics.

S1.1.2 Given a distribution of a variable in a data set, describe its shape, including symmetry or skewness, and state how the shape is related to measures of center (mean and median) and measures of variation (range and standard deviation) with particular attention to the effects of outliers on these measures.

S1.2 Measures of Center and Variation

S1.2.1 Calculate and interpret measures of center including: mean, median, and mode; explain uses, advantages and disadvantages of each measure given a particular set of data and its context.

S1.2.2 Estimate the position of the mean, median, and mode in both symmetrical and skewed distributions, and from a frequency distribution or histogram.

S1.2.3 Compute and interpret measures of variation, including percentiles, quartiles, interquartile range, variance, and standard deviation.

S1.3 The Normal Distribution

S1.3.1 Explain the concept of distribution and the relationship between summary statistics for a data set and parameters of a distribution.

S1.3.2 Describe characteristics of the normal distribution, including its shape and the relationships among its mean, median, and mode.

S1.3.3 Know and use the fact that about 68%, 95%, and 99.7% of the data lie within one, two, and three standard deviations of the mean, respectively in a normal distribution.

S1.3.4 Calculate z-scores, use z-scores to recognize outliers, and use z-scores to make informed decisions.

STANDARD S3: SAMPLES, SURVEYS, AND EXPERIMENTS

S3.1 Data Collection and Analysis

S3.1.1 Know the meanings of a sample from a population and a census of a population, and distinguish between sample statistics and population parameters.

S3.1.2 Identify possible sources of bias in data collection and sampling methods and simple experiments; describe how such bias can be reduced and controlled by random sampling; explain the impact of such bias on conclusions made from analysis of the data; and know the effect of replication on the precision of estimates.

S3.1.3 Distinguish between an observational study and an experimental study, and identify, in context, the conclusions that can be drawn from each.

STANDARD S4: PROBABILITY MODELS AND PROBABILITY CALCULATION

S4.1 Probability

S4.1.1 Understand and construct sample spaces in simple situations (e.g., tossing two coins, rolling two number cubes and summing the results).

S4.1.2 Define mutually exclusive events, independent events, dependent events, compound events, complementary events and conditional probabilities; and use the definitions to compute probabilities.

S4.2 Application and Representation

S4.2.1 Compute probabilities of events using tree diagrams, formulas for combinations and permutations, Venn diagrams, or other counting techniques.

S4.2.2 Apply probability concepts to practical situations, in such settings as finance, health, ecology, or epidemiology, to make informed decisions.

RECOMMENDED:

***S3.1.4** Design simple experiments or investigations to collect data to answer questions of interest; interpret and present results.

***S3.1.5** Understand methods of sampling, including random sampling, stratified sampling, and convenience samples, and be able to determine, in context, the advantages and disadvantages of each.

***S3.1.6** Explain the importance of randomization, double-blind protocols, replication, and the placebo effect in designing experiments and interpreting the results of studies.

***S3.2.1** Explain the basic ideas of statistical process control, including recording data from a process over time.

***S3.2.2** Read and interpret basic control charts; detect patterns and departures from patterns.

***S4.1.3** Design and carry out an appropriate simulation using random digits to estimate answers to questions about probability; estimate probabilities using results of a simulation; compare results of simulations to theoretical probabilities.

NOTES

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